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Technical Note

Effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow

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Abstract

The effect of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field has been studied in the case of unsteady flow. The plate is moved with a constant velocity, which is in the same or opposite direction to the free stream velocity. The fluid viscosity is assumed to vary as an inverse linear function of temperature. The effect of the induced magnetic field has been included in the analysis. The governing partial differential equations have been solved numerically using the finite difference method. The velocity, the x-component of the induced magnetic field and heat transfer characteristics of the flow are determined

Keywords: Free convection; Radiation; Variable viscosity; Aligned magnetic field

1. Introduction

The theory of laminar boundary layer flows on a moving surface occur in several engineering applications. Aerodynamic extrusion of a plastic sheet, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread travelling between a feed roll and a wind-up roll, are examples of practical applications.

Elbashbeshy [2] investigated heat transfer over a stretching surface with variable and uniform surface heat flux subjected to injection and suction. Vayjravelu et al. [3] studied the convective heat transfer in an electrically conducting fluid near an isothermal stretching sheet and they studied the effect of internal heat generation or absorption. Chamkha [4] studied the problem of steady, laminar, free convection flow over a vertical porous surface in the presence of a magnetic field and heat generation or absorption.

All the above investigations are restricted to MHD flow and heat transfer problems. However, of late the radiation effect on MHD flow and heat transfer problems have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and a knowledge of radiation heat transfer becomes very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially for the fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity (see [5]). Since the radiation and the variable viscosity are quite complicated, many aspects of its effect on free convection flows

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Nomenclature

a	constant
C_p	specific heat at constant pressure
$e_{b\lambda}$	Plank's function
F	radiation parameter
g	acceleration due to gravity
G_r	Grashof number
H_0	constant magnetic field
H_1, H_2	induced magnetic field components along and perpendicular to the plate
k_λ	absorption coefficient
L	the length of the plate
Pr	Prandtl number
q_r	radiative heat flux
T	temperature
t	time
T_r	constant
T_w	wall temperature
t^*	dimensionless time
T_∞	free stream temperature
u	velocity component in the x-direction
u_∞	free stream velocity
v	velocity component in the y-direction
x, y	distance along and perpendicular to the plate

Greek symbols

α	thermal diffusivity
α_1	magnetic diffusivity
β	magnetic force number
β_v	coefficient of volume expansion
γ	thermal property of the fluid
η	dimensionless co-ordinate
ν	kinematic viscosity
λ	reciprocal of the magnetic Prandtl number
ρ	density
μ_0	magnetic permeability
ε	ratio of the wall to free stream velocities
θ_r	constant
σ	electrical conductivity
ψ	stream function

Subscripts

e	condition at the edge
w	condition at the surface

past a semi-infinite flat plate in the presence of an aligned magnetic field have not been studied in the case of unsteady flow. Hence, we propose investigating the radiation and variable viscosity effects on unsteady free convection flows in the presence of an aligned magnetic field. The governing partial differential equations have been solved numerically using the finite difference method [6]. The velocity, the x-component of the induced magnetic field, and heat transfer characteristics of the flow are calculated.

2. Mathematical formulation

Consider the unsteady laminar, incompressible, viscous, electrically conducting fluid flowing past a fixed semi-infinite plate having a constant free stream velocity u_∞. The fluid is considered to be a gray, absorbing or emitting radiation, but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The fluid has ρ, ν, α, μ_0 and α_1 all these quantities are supposed to be constant. A constant magnetic field H_0 is applied parallel to the plate outside the boundary layer. We assume that the normal component of the induced magnetic field H_2 vanishes at the wall and the parallel component H_1 approaches the given value H_0 at the edge of the boundary layer. The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity which is assumed to be an inverse linear function of temperature (see [1]).

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] \quad \text{or} \quad \frac{1}{\mu} = a(T - T_r),$$

where $a = \gamma/\mu_\infty$ and $T_r = T_\infty - 1/\gamma$.

Both a and T_r are constant, and their values depend on the reference state and the thermal property of the fluid, i.e., γ. In general $a > 0$ for liquids and $a < 0$ for gases. We further assume that the Ohmic dissipation term and the Hall effects have been neglected. The wall temperature T_w and the free stream temperature T_∞ are taken as being constant.

Under the usual boundary layer approximation, the flow and heat transfer in the presence of radiation and variable viscosity are governed by the following equations [7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\mu_0}{\rho}\left[H_1\frac{\partial H_1}{\partial x} + H_2\frac{\partial H_1}{\partial y}\right], \tag{3}$$

$$\frac{\partial H_1}{\partial t} + u\frac{\partial H_1}{\partial x} + v\frac{\partial H_1}{\partial y} - \frac{\partial u}{\partial x} - H_2\frac{\partial u}{\partial y} = \alpha_1\frac{\partial^2 H_1}{\partial y^2}, \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}. \tag{5}$$

The boundary conditions are given by

$$u = u_w,\ \ v = H_2 = \frac{\partial H_1}{\partial y} = 0,\ \ T = T_w \quad \text{at } y = 0,$$
$$u \to u_\infty,\ \ H_1 \to H_0,\ \ T \to T_\infty \quad \text{as } y \to \infty. \tag{6}$$

Here H_0 is the applied magnetic field parallel to the x-axis in the free stream, H_1 and H_2 are the components of the induced magnetic field in the x and y directions, respectively. Greif et al. [8] followed closely the analysis of Cogley et al. [9] who showed that, for an optically thin limit, the fluid does not absorb its own emitted radiation, i.e., there is no self-absorption, but the fluid does absorb radiation emitted by the boundaries. Cogley et al. [9] showed that, in the optically thin limit for a gray-gas near equilibrium, the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4(T - T_w)I,$$

where

$$I = \int_0^\infty k_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w \mathrm{d}\lambda.$$

In order to reduce the number of independent variables from three to two and to make the governing equations dimensionless, we apply the following transformations:

$$\eta = \left[\frac{u_\infty}{v}\right]^{1/2} x^{-1/2} y, \quad u = u_\infty f'(\eta, t^*),$$
$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x},$$
$$\psi = (u_\infty v)^{1/2} y^{1/2} f(\eta, t^*), \quad \lambda = \frac{\alpha_1}{v},$$
$$\theta(\eta, t^*) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi = H_0\left(\frac{vx}{u_\infty}\right)^{1/2} g(\eta, t^*),$$
$$G_r = \frac{g\beta_v L^3 \Delta T}{v^2}, \tag{7}$$
$$F = \frac{4IL^2}{\rho C_p v\sqrt{G_r}}, \quad t^* = u_\infty t/x,$$
$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)},$$
$$H_1 = \frac{\partial \varphi}{\partial y}, \quad H_2 = -\frac{\partial \varphi}{\partial x},$$
$$H_1 = H_0 g'(\eta, t^*), \quad \beta = \mu_0 H_0^2/\rho u_\infty^2, \quad \varepsilon = \frac{u_w}{u_\infty} < 1.$$

The prime denotes derivatives with respect to η.

Then Eqs. (1)–(5) become

$$2f''' - 2\left(\frac{1}{\theta - \theta_r}\right)\theta' f'' + \left(\frac{\theta - \theta_r}{\theta_r}\right) \times \left\{\beta g g'' - f f'' + 2\frac{\partial f'}{\partial t^*} - 2t^*\left[f'\frac{\partial f'}{\partial t^*} - \frac{\partial f}{\partial t^*} f''\right]\right\} + 2\beta t^*\left(\frac{\theta - \theta_r}{\theta_r}\right)\left[g'\frac{\partial g'}{\partial t^*} - g''\frac{\partial g}{\partial t^*}\right] = 0, \tag{8}$$

$$2\lambda g''' + f g'' - f'' g - 2\frac{\partial g'}{\partial t^*} + 2t^*\left[f'\frac{\partial g'}{\partial t^*} - \frac{\partial f}{\partial t^*} g''\right] - 2t^*\left[g'\frac{\partial f'}{\partial t^*} - f''\frac{\partial g}{\partial t^*}\right] = 0, \tag{9}$$

$$\frac{2}{Pr}\theta'' + f\theta' - 2F(\theta - 1) - 2\frac{\partial \theta}{\partial t^*} + 2t^*\left[f'\frac{\partial \theta}{\partial t^*} - \frac{\partial f}{\partial t^*}\theta'\right] = 0. \tag{10}$$

The boundary conditions are given by

$$f(0,t^*) = g(0,t^*) = g''(0,t^*) = \theta(0,t^*) = 0,$$
$$f'(0,t^*) = \varepsilon, \quad f'(\infty,t^*) = g'(\infty,t^*) = \theta(\infty,t^*) = 1 \tag{11}$$

and the initial conditions are given by the steady-state equations, obtained by putting $t^* = 0$ and $\partial/\partial t^* = 0$ in Eqs. (8)–(10), where β is the magnetic force number, which is the square of the ratio of the Alfven speed to the free stream velocity, and λ is the reciprocal of the magnetic Prandtl number, which is the ratio of the viscous and the magnetic diffusivity.

We have solved the parabolic partial differential equations (8)–(10) under the boundary conditions (11) and the initial conditions by using an implicit, iterative, tridiagonal finite-difference method similar to that discussed by Blottner [6]. These equations are integrated by a shooting method, fourth-order Runge–Kutta, with step size 0.01. It is worth mentioning here that for $\gamma \to 0$; i.e., $\mu = \mu_\infty$ (constant) then $\theta_r \to \infty$ and Eq. (8) reduces to that of Takhar et al. [7]. Also, when $F = 0$ then Eq. (10) reduces to the same reference. It is also important to note that θ_r is negative for liquids and positive for gases, when $(T_w - T_\infty)$ is positive, see Lai and Kulacki [1]. It is observed here that radiation and variable viscosity does affect the velocity and temperature field of free convection flow of an electrically conducting fluid. The velocity component $f'(\eta, t^*)$ and the x-component of the induced magnetic field $g'(\eta, t^*)$ as well as the temperature $\theta(\eta, t^*)$ distribution are presented in Figs. 1–3 for various values of F, θ_r, and λ at $\varepsilon = 0.1$ and $t^* = 0.01$.

3. Results and discussion

To study the behavior of the velocity, the x-component of the induced magnetic field and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

Fig. 1(a) shows that $f'(\eta, t^*)$ decreases with increasing the viscosity parameter θ_r. But the temperature $\theta(\eta, t^*)$ increases as the viscosity parameter θ_r increase as seen in Fig. 1(b) The results presented demonstrate quite clearly that θ_r, which is an indicator of the variation of viscosity with temperature, has a substantial effect of the drag and heat transfer characteristics as well as the velocity and

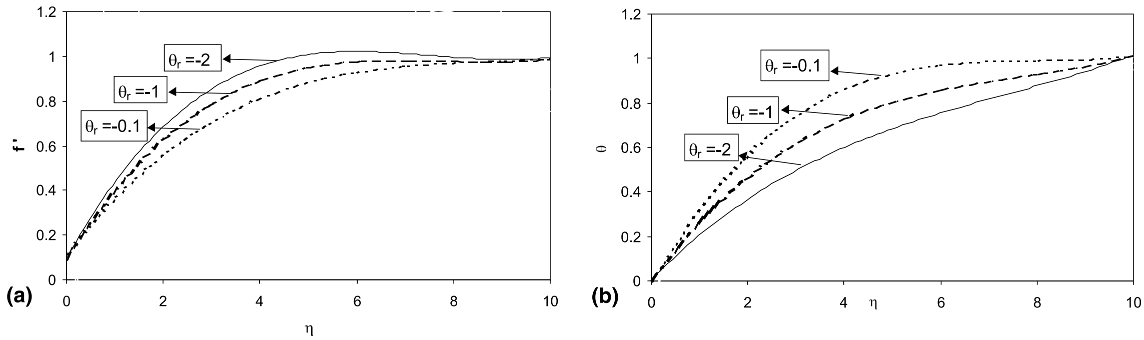

Fig. 1. The effect of the viscosity parameter θ_r on: (a) the velocity profiles $f'(\eta, t^*)$ and (b) the temperature profiles $\theta(\eta, t^*)$, for $Pr = 0.7$, $\lambda = 10$, $\beta = 0.5$ and $F = 3$.

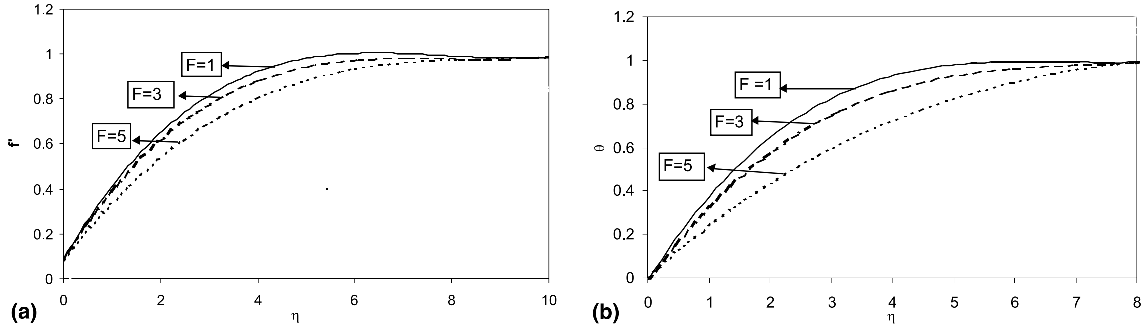

Fig. 2. The effect of the radiation parameter F on: (a) the velocity profiles $f'(\eta, t^*)$ and (b) the temperature profiles $\theta(\eta, t^*)$, for $Pr = 0.7$, $\lambda = 10$, $\beta = 0.5$ and $\theta_r = -0.1$.

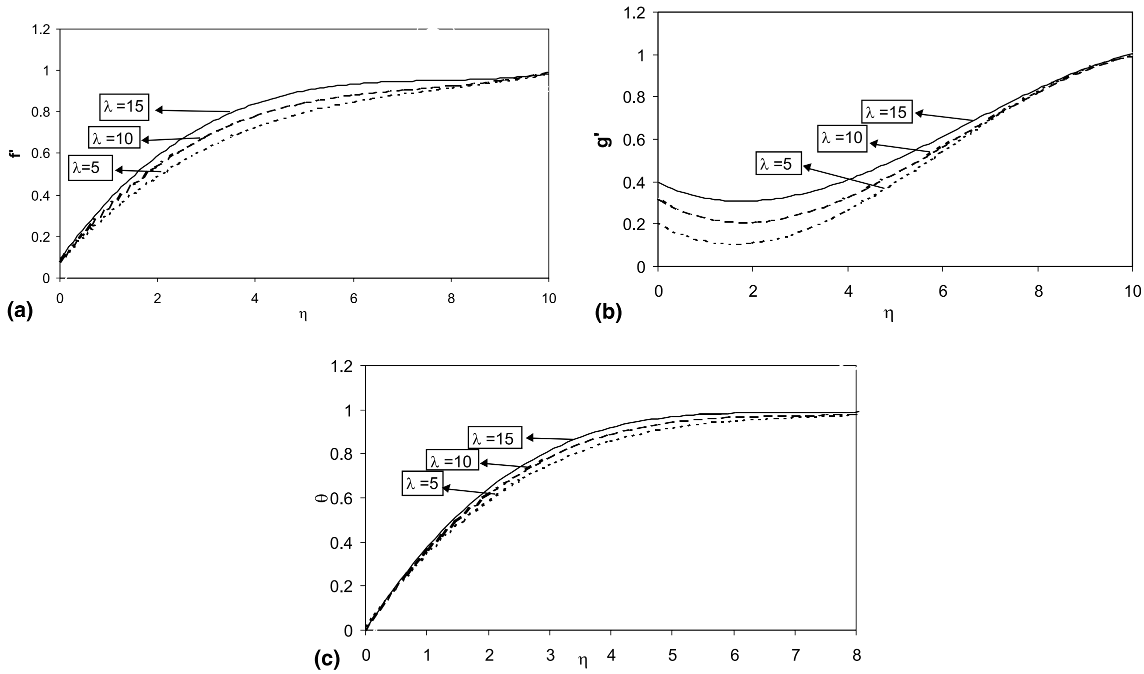

Fig. 3. The effect of reciprocal of the magnetic Prandtl number λ on: (a) the velocity profiles $f'(\eta, t^*)$, (b) the induced magnetic field profiles $g'(\eta, t^*)$ and (c) the temperature profiles $\theta(\eta, t^*)$, for $Pr = 0.7$, $\beta = 0.5$, $F = 3$ and $\theta_r = -0.1$.

temperature distributions within the boundary layer over a continuous moving flat plate.

Fig. 2(a) shows that $f'(\eta, t^*)$ decreases with increasing the radiation parameter F. It is seen from Fig. 2(b) that the temperature $\theta(\eta, t^*)$ decreases as the radiation parameter F increases. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

Fig. 3 present the effect of the reciprocal of the magnetic Prandtl number λ on $f'(\eta, t^*)$, $g'(\eta, t^*)$ and $\theta(\eta, t^*)$. It is observed from these profiles that the effect of λ is more pronounced on $g'(\eta, t^*)$ and its effect on $\theta(\eta, t^*)$ is very small. This is because λ occurs in the equation for the induced magnetic field and its effect on $\theta(\eta, t^*)$ is indirect. The effects of magnetic force parameter β and the time t^* have been studied in [7].

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